METHODS OF DATA REDUCTION FOR THE OPEP AIRGLOW PHOTOMETER ON OGO-II

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Preliminary Report

This work was performed while the author was at the Goddard Space Flight Center, Greenbelt, Maryland, U.S.A., in connection with the Airglow Photometer Experiment on OGO-II.

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TABLE OF CONTENTS

		Page
INTRODUCTION		1
PROBLEMS REVEA	ALED BY "QUICK LOOK" ANALYSIS	1
CALIBRATION OF	THE PHOTOMETER	3
REDUCTION OF T	THE OSCILLATIONS	3
DIFFERENT PROG	FRAMS OF THE DATA REDUCTION	4
FIRST PROGRAM		8
MANUAL PROCESS	SING OF THE DATA	14
SECOND PROGRAM	I	14
THIRD PROGRAM		15
GENERAL COMMENTS		
	JSE OF DATA FROM A SATELLITE-BORNE	22
APPENDIX B: O	PRIENTATION OF THE OPEP	27
APPENDIX C: R	REDUCTION OF OSCILLATIONS	35
	COMPUTATION OF VERTICAL EMISSION PROFILE	41

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ILLUSTRATIONS

		Page
1.	First Program	5
2.	Second and Third Programs	6
3.	Plot of one scan of data	12
4.	Field of view of the photometer	13
5.	Vertical emission profile, single scan method	17
6.	Lines of sight used in parallel method	19
7.	Vertical emission profile, parallel method	20
App	endix B	
1.	Coordinate system used in Method No. 1	28
2.	Definition of Ia	29
3.	Coordinate system used in Method No. 2	32
4.	Coordinates of the star	33
App	endix C	
1.	Stray light	36
2.	Data showing the oscillations	39
3.	Same data after processing by this method	40
App	endix D	
1.	Geometry of the problem in the plane of the orbit	43
2.	Intensity as a function of altitude	45
3.	Criteria for selecting layers	51

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JACQUES PACQUET

ABSTRACT

A brief description is given of the methods used in the reduction of the data from the OPEP airglow photometer on OGO-II. For selected portions of the data, computers have been used to apply instrumental corrections, to reduce an oscillatory component dependent on the position of the OPEP container, and to solve a system of linear equations to compute a vertical emission profile from the observed horizon profiles.

INTRODUCTION

This preliminary report is intended to give a better understanding of the problems encountered during the data reduction of the Airglow Photometer experiment on OGO-II. In particular, the solutions used to solve some of these problems are described in detail in the appendixes.

The different steps of the data reduction of the data will not be described in detail. A knowledge of the experiment, the spacecraft and its telemetry, and the format of the data furnished to the experimenter is assumed. The programs in detail with flow charts and instructions for use will be described in a later document.

The approach to the problems of satellite data reduction and analysis is described in general terms in Appendix A. Since the volume of useful data from OGO-II is relatively small, emphasis has not been placed on the development of a system capable of efficiently handling hundreds of reels of tapes. First a "quick look" analysis was made to identify the problems that developed in the OGO-II data. Secondly, a set of programs has been developed and used to process the approximately 30 hours of data that were available when the spacecraft was in a stable attitude with respect to the earth.

PROBLEMS REVEALED BY "QUICK LOOK" ANALYSIS

During the first few months of data analysis, the data was examined by means of listings of the raw data on the decomm data tape, listings and strip charts prepared by the OGO Control Center, and plots, made both manually and automatically.

The spacecraft went into a higher orbit than anticipated and did not maintain the stable attitude that was expected. These, and reflections from the spacecraft gave rise to a number of problems such as:

1. Because of the high apogee, during many scans, the earth's horizon was below the field of view of the photometer. There was little or no airglow data of value when the spacecraft was above 1200 km.

- 2. The high apogee resulted in a higher dose of radiation than expected, with the result that the output of the high voltage power supply drifted out of the range of its monitor. After the first day, he level of high voltage, and hence the gain of the photomultiplier, could no longer be determined from the high voltage monitor.
- 3. There was a high level of stray light, the principal source of which appeared to be reflections from the VLF antenna on one of the long booms of the spacecraft. The level of light was dependent on the azimuth of the OPEP container, the mirror position, and the positions of the sun and sunlit earth. The level of stray light had to be evaluated independently for each scan. When the earth was sunlit, the photometer data was of no value for airglow measurements.
- 4. The OPEP container constantly "dithered" about its intended position at a rate of about 0.5 cps. This resulted in a modulation of the stray light entering the photometer and hence a corresponding oscillation in the data. A method was found to remove this oscillation from part of the data.
- 5. During those occasions (at least several times an orbit) that the spacecraft attitude control system failed to orient the spacecraft properly, the direction in which the photometer is looking is unknown, and the data is of no value.
- 6. Two stars, Sirius and Canopus, were observed a number of times. This made possible a calibration of the sensitivity of the photometer and a check on the relationship between the photometer field of view and the axes of the spacecraft as defined by the attitude control system.

CALIBRATION OF THE PHOTOMETER

It was noted that there was a signal corresponding to the star, Canopus, in the data from almost all of the orbits in which there was useful information. This star was used to evaluate the sensitivity of the photometer.

To do this, the star was treated as if it were an extended source. Data from orbit numbers 8 and 9, before the output of the high voltage power supply had drifted outside the range of the high voltage monitor, were used. By using the ground calibrations of the photometer, and making appropriate adjustments for temperature and the measured value of the high voltage power supply, the amount of light from Canopus was expressed in rayleighs.

For each orbit thereafter, the signal from Canopus was used to obtain the sensitivity of the photometer in terms of rayleighs per volt. These values were entered as input parameters for the main program of data reduction.

These stars were also used to check the orientation of the OPEP photometer as given by the attitude-orbit tape. In Appendix B are given the details of the methods used to compute the apparent location of the stars as given by their appearance in the photometer. It was found that this apparent location varied by about 2-1/2 degrees, from negative values on October 15, 1965 to a positive value on October 23, 1965. The altitude of the lower of the two airglow layers was also computed, and at all times was between 60 and 113 km. This is equivalent to an angular variation of 1.3 degrees and was more nearly random as a function of time. Errors in either or both the Universal Time associated with the photometer data in the decomm data tape or the orbit given by the attitude-orbit tape could be responsible.

REDUCTION OF THE OSCILLATIONS

Examination of the data from the OPEP photometer led to the following hypotheses:

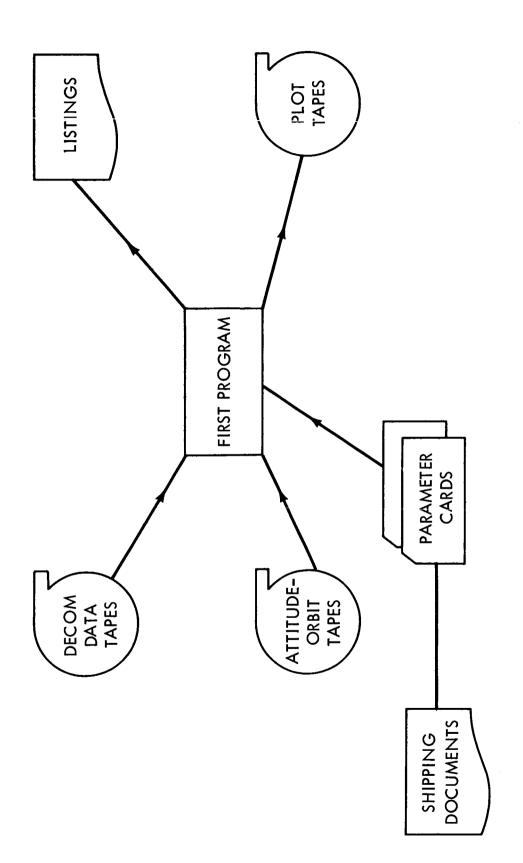
- 1. Most of the stray light entering the instrument has been reflected from the spacecraft.
- 2. The level of this stray light is a function of the azimuth angle of the OPEP container and of the mirror position of the photometer.
- 3. The 0.5 cps oscillation noted in the output of the photometer is due to the motion of the OPEP container about its intended azimuth such that it receives varying amounts of stray light.

After trying several methods of reducing the oscillation in the data, including various filtering techniques, it was found that best results were obtained by applying a correction to the data which is a function of the azimuth angle of the OPEP container as indicated by housekeeping data from the spacecraft. The data were considered one scan at a time, noisy data were identified and deleted, and an average value of azimuth determined for that scan. Several oscillations were examined to obtain a value for their amplitude, and then appropriate corrections were applied to all the data in that scan. Further details are given later in the description of the first program and in Appendix C.

This method is superior to filtering techniques in that it tends to preserve the high frequency aspects of interest, such as stars and steep gradients in the airglow profile. Because the low scale had an additional component of noise from some electronic source, no positive results could be obtained when the photometer output was in the region between about 4.5 and 15 volts. However, the data in about 30% of the scans have been improved by the application of this method.

DIFFERENT PROGRAMS OF THE DATA REDUCTION

After the preliminary examination of the data, the reduction was divided into three main parts, as diagrammed in Figs. 1 & 2. The first program used the decommutated data tape, and applied a number of adjustments and corrections that were found to be necessary in order to determine the



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Figure 1 - Block diagram of the First Program

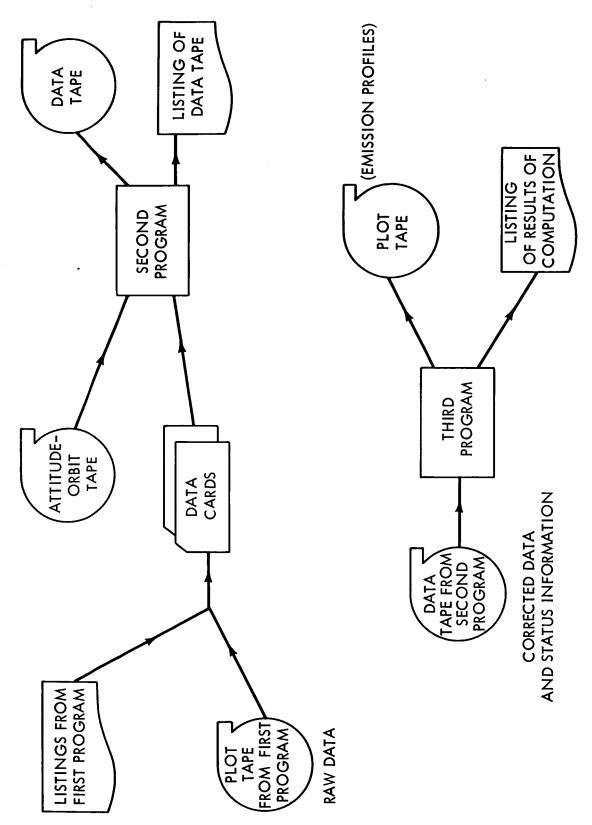


Figure 2 - Block diagrams of the Second and Third Programs

total amount of light entering the instrument. This was merged with appropriate data from the attitude-orbit tape and the results were listed and plotted.

The plots from the first program are examined and an estimate made (manually) of the level of stray light during each scan. The amount of light attributed to airglow is punched on cards and is used as input to the second program. The second program merges these data with pertinent data from the attitude-orbit tape, and prepares a data tape.

The third program enters these data in a set of simultaneous linear equations, and solves the set to obtain the emission rate of the airglow as a function of altitude.

FIRST PROGRAM

The result of processing by this program is a measure of the light entering the photometer. The inputs are the decomm data tape and the attitude-orbit tape. The outputs are a plot tape and a listing. In processing, questionable data points have been deleted and appropriate scale factors have been applied. The data have been considered in terms of blocks, each containing the data for one mirror scan and where feasible, the oscillations due to the motion of the OPEP container have been reduced. The results have been plotted along with selected information from the attitude-orbit tape. The following sections describe these steps in more detail.

Selection of data

The data to be processed is specified by the initial and end times $(T_1 \text{ and } T_2)$ desired. In order to select the reels and files containing the data between T_1 and T_2 , the shipping documents prepared by Information Processing Division are examined. The times, T_1 and T_2 are punched on a card which is introduced at the time of the run. The program uses T_1 to position the decomm data tape and the attitude-orbit tape at the beginning of processing, and to stop processing at the time T_2 .

More than one decomm data tape may be processed in a single run, but the data must be in chronological order. However, the addition of a routine to reposition and search the attitude-orbit tape would remove this restriction.

Only good data is used, that is, data for which the digital word indicates that the photometer is not in a calibration, stand-by or some abnormal mode, but is in the proper mode for measuring airglow. If any of the data words in a given frame were off scale, it was assumed that noise was present, and that frame of data was discarded.

Application of calibration factors

After a frame of potential airglow data has been selected, the data is converted to a number corresponding to the amount of light entering the instrument, assuming the total signal is from a diffuse extended source with the emission given in rayleighs.

The steps involved in the conversion are as follows:

- 1. Selection of the most sensitive output word which is on scale.
- 2. Subtraction of the appropriate electrometer zero. Since an examination of samples of the calibration cycle data indicated that drift in the zero level of the electrometer was negligible, a constant value was entered for each of the three linear channels.
- 3. Conversion to electrometer output. A constant was used for each of the three linear words.
- 4. Conversion to emission rate in rayleighs. As determined earlier, after an examination of the response to Canopus, and noting the slowness of the temperature variations, the sensitivity of the photometer was essentially constant for each orbit. The appropriate values were entered by means of punched cards when setting up the computer run.

In general, the dark current was much smaller than the current due to stray light. Hence, it was decided not to subtract dark current in the First Program, but to subtract it at a later stage, when subtracting the assumed level of stray light.

Reduction of oscillations

After the calibration factors have been applied to a block of data, the program examines the digital word to find the end points of the mirror scan, namely mirror positions 0 and 59. The data is then processed one scan at a time (between mirror position 0 and the reversal of mirror travel) in order

to reduce the 0.5 cps component present in the data.

The steps involved include:

- 1. The average azimuth of the OPEP container is found by taking an arithmetic average of the azimuth angle given by the telemetered sine and cosine data.
- 2. Pairs of data points are selected such that for a given mirror position, one point corresponds to the average azimuth for the OPEP container. The deviation which corresponds to the other point is computed.
- 3. The amplitude of the deviation as a function of mirror position is represented by a polynomial of the first degree given by a least square fit of the deviations computed in the preceding step.
- 4. Each data point in the scan is corrected for the deviation as given by the above line.

Further details of the basis of this method are given in Appendix C.

Selection of Attitude-Orbit Information

Before processing a block of data, selected attitudeorbit parameters for the time period to be covered are
entered in core. Data interpolated from these tables are
included in the plots and listings of the processed photometer
data. The interpolation is linear. For the initial and
end times of each mirror scan, the following parameters are
included:

Universal time

Local time of subsatellite point
Geographic latitude of subsatellite point
Geographic longitude of subsatellite point
Geomagnetic latitude of subsatellite point
Height of satellite

Plots

This First Program creates a plot tape for the Stromberg-Carlson 4020 plotter such that each plot frame contains the data pertaining to a single scan of the mirror. An

example of a plot frame is given in Figure 3. The data, generally 2 points per mirror position, are plotted in volts as a function of mirror position. The time, mirror position, and pertinent attitude-orbit information for the start and end time of the scan are tabulated near the top, along with the conversion factor to rayleighs.

At the top of the grid is printed a number for each mirror position which corresponds to the altitude, h, of the minimum height of the mean direction of the line of sight of the photometer (See Figure 4). When the line of sight is above or at the maximum of the airglow layer, h is an indication of altitude from which originates the major contribution to the observed airglow.

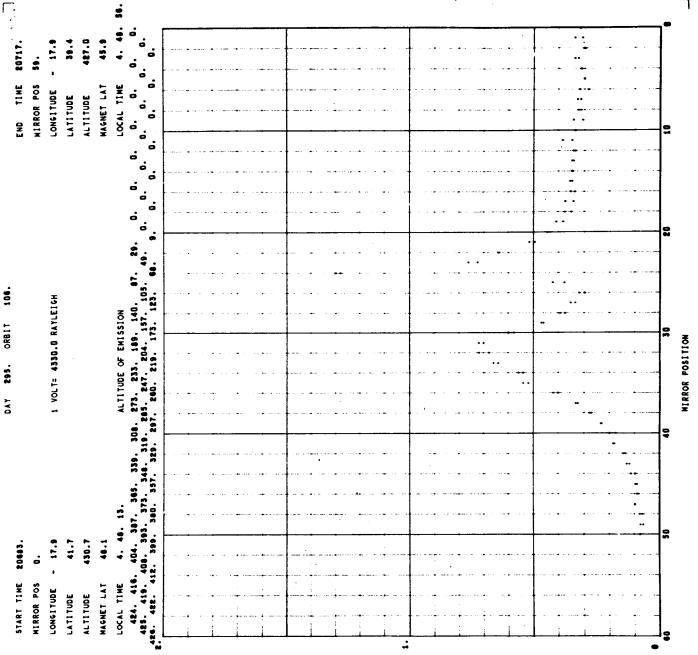
The value for h is computed from

 $h = (r + zs) \cos \alpha - r$

where r = radius of the earth (assumed an average value of 6370 km)

zs = height of the spacecraft above the earth (interpolated value)

 α = angle of the line of sight



>0J-0

a 4020 plot of the data in one mirror - Example of scan. Figure 3

PHOTOMETER IN SPACECRAFT 11

ALTITUDE OF EMISSION
ALTITUDE OF SPACECRAFT
SIGHT ANGLE
RADIUS OF EARTH

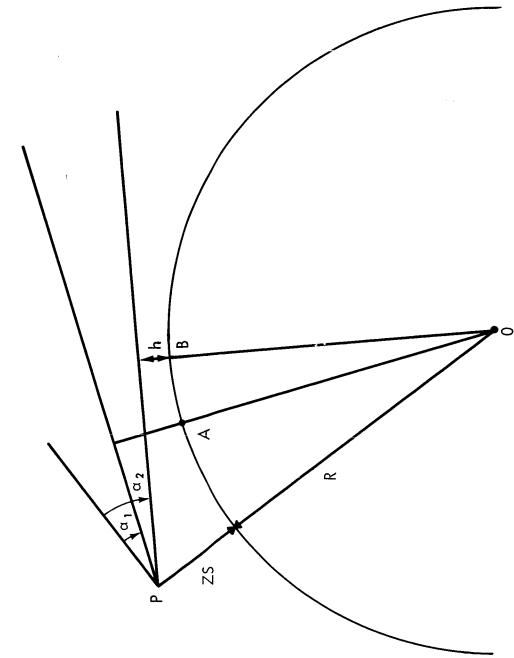


Figure 4 - Location of the field of view of the photometer with respect to the earth.

MANUAL PROCESSING OF THE DATA

The plotted data from the First Program were examined manually in order to discard noise, identify stars, estimate stray light, and finally to estimate the signal due principally to airglow.

Those scans which were excessively noisy or had a large number of missing points were discarded. The star signals were used in checking the orientation of the photometer. A smooth curve was drawn by hand through the data points with an attempt to preserve features of the signal and yet to distinguish the airglow information from stray points. It was then assumed that the stray light could be represented by a straight line. Using principally the information at low numerical values of mirror position, a line was drawn for each scan. The difference between the curve through the data points and the straight line representing stray light was measured. Later these differences were punched on cards along with the start time of the scan, the end time, and the number of data points.

SECOND PROGRAM

The Second Program was written for the purpose of creating a data tape from the punched card data resulting from the manual processing. The new data tape would also contain appropriate attitude-orbit information.

After positioning the attitude-orbit tape, the program stores tables of attitude-orbit information. The data needed for the output tape is obtained by linear interpolation.

The first record of each file in the output data tape is an identification record. The second is a copy of the table of stored attitude-orbit information. Each of the following records contain the photometer and attitude-orbit data for one mirror scan. A separate file is generated for each orbit. A double end of file is generated after the last file or last orbit of data.

Each data record in the file includes for each data point:

- 1. Universal time
- 2. The value of the data point in volts
- 3. The direction of the line of sight (mirror position)
- 4. The magnitude of the position vector
- 5. The true anomaly of the satellite
- 6. The height of the satellite

When an update attitude-orbit tape is received, it can be merged with the photometer data by the use of this program to generate a new analysis tape.

THIRD PROGRAM

The purpose of this program is to compute the emission rate of the airglow as a function of altitude, using the photometer data and attitude-orbit data on the tape from the Second Program. The resulting profile is displayed by the 4020 plotter.

Two versions are available. The first uses the data from a single scan to the mirror. The second was written in order to better define the location of the emission profile, and uses selected data from several mirror scans. Each version also computes data relative to the location of the emission profile.

Single scan method

The data from one scan of the mirror is used. To avoid the effects of absorption and refraction by the earth and the lower atmosphere, data corresponding to lines of sight with minimum altitudes (h) of less than 50 km are discarded. (see Figure 4).

Certain hypotheses have been assumed for the computation of the emissions versus altitude:

1. The earth in the locality under consideration is assumed to be a sphere. The radius, r, of this sphere is given by:

$$R^{2} = \frac{a^{2}(f-1)^{2}}{(f-1)^{2}\cos^{2}\varphi + f^{2}\sin^{2}\varphi}$$

- where a = semi-major axis of the spheroid representing the earth,
 - f = flattening which is a/(a b), b is the semi-minor axis, and
 - ϕ = latitude of the average sub-satellite point during the scan of the photometer.
- 2. The airglow is represented as a number of thin, uniform, spherical layers. For each scan all but the initial layer are chosen to have the same thickness, a minimum of 15 km. The contribution of each layer to the observed emission rate along each line of sight of the various mirror positions can be represented by a set of simultaneous linear equations. The computations of the unknowns representing the emission of each thin layer is done by a least squares method. The details of this and the method of solution are given in Appendix D.

An example of the results of this method is shown in the 4020 plot reproduced in Figure 5. In this example, the emission rates were computed five times, each time using a different thickness for the initial layer.

The tabular data at the top of the plot indicate the time and location of the profile. This is done by giving data pertinent to positions A and B (see Figure 4) for the largest and smallest sight angles used in this computation. This is done by adding the angle of the line of sight (α_1 or α_2) to the true anomaly of the spacecraft at the time of the observation. Using the result as a true anomaly, interpolation is made in the data of the table of attitude-orbit data to find the subsatellite point corresponding to that angle. The longitude of this point is corrected for the rotation of the earth. Similarly, the local mean time, geomagnetic time, and the geomagnetic coordinates are also computed.

PHOTONS/CM·3/SEC

Figure 5 - Emission rate as a function of altitude, 4020 plot of results of computation. Location tables are for the first and last data points selected. (Points A and B of Fig. 4).

Parallel method

This method uses data selected from several successive scans of the mirror and is used for the purpose of computing the emission over specified locations. From the several mirror scans, data is selected for which the line of sight is perpendicular (or nearly so) to the local vertical through the selected location. See Figure 6. As in the single scan version, no data is used for which the line of sight is less than 50 km from the surface of the earth.

The hypotheses for the computations are similar to those for the single scan method except that in the computation of R, ϕ is the latitude of the selected location. Also, in order to use the same routines for the computations, it was found convenient to compute an apparent position and line of sight for each data point, as indicated in Figure 6.

An example of the result of this method, chosen for about the same time and location as in Figure 5, is shown in Figure 7. The similarity of the two is a measure of the internal consistency of the experiment data.

Here also, tabular data have been given to describe the time and location of the emission profile. When the parallel method is used, the airglow layers are not chosen to be of equal thickness. The emission profile in Figure 7 was computed twice: once with a minimum layer thickness of 12 km, and again with the layers selected in a different manner but with a minimum thickness of 20 km.

General Comments

All programs have been written partly in MAP (input routines) and partly in Fortran IV for the 7094 Model II. Plot tapes for the Stromberg-Carlson 4020 plotter are generated through the use of the North American software package.

Besides these three programs designed for production processing of the data, several other special purpose programs have been completed. Examples include the computation of the location of the stars, Canopus and Sirius, as observed by the photometer, and computation of the emission profile

P_i DIFFERENT POSITIONS OF THE PHOTOMETER
A POINT OVER WHICH EMISSION IS TO BE COMPUTED
P'_i APPARENT POSITIONS OF THE PHOTOMETER
α'_i APPARENT SIGHT ANGLES

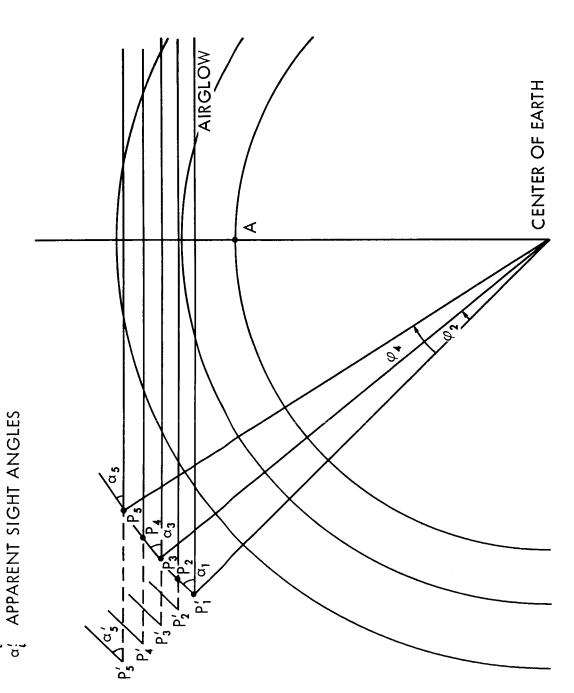


Figure 6 - Lines of sight used in parallel method.

ORBIT 105. YEAR 1965. DAY 295. UNIVERSAL TIME 14011. GEOGRAPHIC LONG 8.7 GEOGRAPHIC LAT 52.7 GEOMAGNETIC LONG 94.1 GEOMAGNETIC LAT LOCAL TIME 4. 28.6 GEOMAGNETIC TIME 82.5

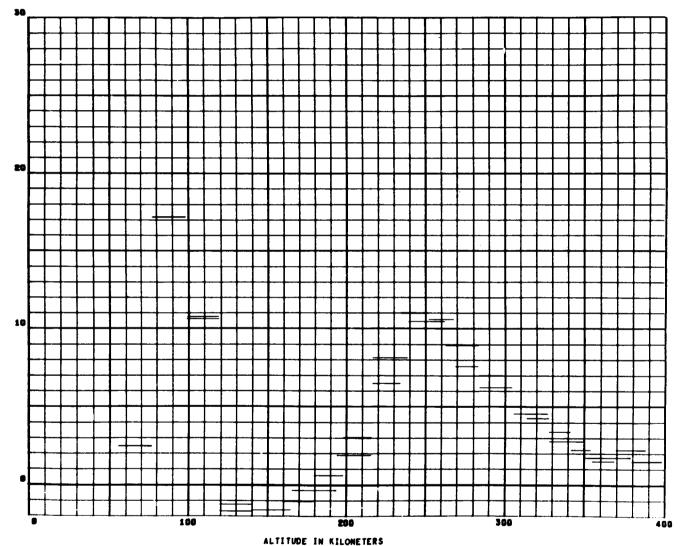


Figure 7 - Emission rate as a function of altitude, 4020 plot of results of computation. Location table is for point A of Fig. 6.

by representing it as a polygonal line.

Even in this set of three programs for production processing, it is obvious that many of the functions are elementary and need improvement. It is hoped that for the future these problems will have a better solution.

APPENDIX A

USE OF THE DATA FROM A SATELLITE-BORNE EXPERIMENT

INTRODUCTION

This paper is not for the purpose of defining a method for using the data from an artificial satellite, but rather is for the purpose of giving a general idea of the problem. It is really difficult to generalize because the method depends principally on the satellite, the volume of information, and the particular experiment. However, it is possible to make some general remarks which are of value for all cases.

PRINCIPAL PROBLEMS

1. <u>Volume of data</u>: The major problem is the volume of data. The principal factors are the lifetime of the satellite, the telemetry capacity, and the proportion of this assigned to the particular experiment.

Consider, for example, the data recorded on board the satellite, OGO II, from experiment 5012. The information rate of the telemetry system is 4 kilobits per second (kbs). Assigned to this experiment are 10 words of 9 bits each, each of which is sampled every 288 seconds. During the process of digitization and decommutation, the amount of information is multiplied by about a factor of 4. For the first 10 days, the experimenter has received 25 digital magnetic tapes containing about 5 to 7 hours on each. For a satellite lifetime of 6 months this corresponds to about 450 tapes. To this number must be added an equal number of tapes which were recorded in real time at data rates of 16 and 64 kbs.

- 2. Quality of the data: Sometimes due to noise, sometimes to a loss in synchronization, the quality and usefulness of the data becomes highly questionable. The problem becomes one of knowing how to recognize these periods and to distinguish the noise from the physical phenomenon. The only method, sometimes, is to list the information for the period in question.
- 3. <u>Time</u>: In order to identify and locate the data, each point is associated with time. This time is sometimes in error due to a malfunction of the clock in the satellite, to noise, or

to the time provided by the tracking stations. It has been noted that for experiment 5012 on OGO II, the Universal Time on "Quick Look" tapes is erroneous for every 8th data point.

Other errors such as the day of the year or even the year occur. Certain of these errors cannot be detected except by visual examination.

4. Duplication of data: Several tracking stations simultaneously record the data from the satellite. Also, some of the data has been digitized several times, and each time, a tape is furnished to the experimenter. Finally, added to these normal problems, are abnormal ones such as malfunctions of the satellite or the experiment which again complicate the use of the data.

APPROACH TOWARD A SOLUTION

One would like to think of immediately applying a program for analysis of the data directly to the tape received by the experimenter. This is not desirable for several reasons:

- 1. A tape contains several files, each corresponding to a passage of the satellite over a tracking station. The decommutation program puts these files in chronological order on the tape, but intervals of time may occur between these blocks of data. The data corresponding to these intervals may be on other tapes with data which have been recorded by another station.
- 2. These tapes contain raw data. Sometimes to give them meaning, this data must be calibrated. If the calibration and analysis are done by the same program, that program becomes overloaded and unmanageable.
- 3. The volume of the data requires a serial catalog of the data, with a description of the tapes and files for the purpose of future utilization.
- 4. The duplication of data requires that criteria for selection of data for analysis be established. These criteria would apply to the data added during decommutation (quality indicators).

All these reasons lead one to define two phases in the use of the data. These two steps are called "Data Reduction" and "Data Analysis".

DATA REDUCTION

The "Data Reduction" is the first passage of the tapes through the computer. This program applies all the operations to the data except those of analysis and generates tapes containing calibrated data which can be used for analysis. It includes operations such as:

- 1. Calibration of the data, that is to say, conversion of the raw data into physical (engineering) units. This step of the programming must be versatile, considering the problems that one can encounter which make that conversion difficult.
- 2. Correction of data affected by noise. This is really the generation of indicators describing the quality of the data. These indicators serve, for example, in the case of duplication of the data, or even in the analysis of the data itself. The corresponding periods could be tagged by a quality indicator.
- 3. Generation of a catalog. This catalog would give as complete a description as possible of the data on the "output" tape. It would be updated during each passage of the program through the machine. This is the fundamental tool for the retrieval of data for a certain period. This catalog would be a basic instrument in the analysis of the data.
- 4. Compression of the data. After calibration, the volume of data is diminished by a factor of about 10. The "output" of the program would be more manageable because of their smaller number. It is not unusual to see almost 600 tapes compressed to 50. It could also be merged with data from other sources, such as the attitude orbit tape of the satellite, without which the experiment data has no significance.
 - 5. To these functions are added listings, plots, etc.

This enumeration cannot be complete because certain functions depend wholly on the experiment itself. However, the logic of the program is absolutely general and should be useful for each experiment.

The second part depends entirely on the experimenter and the experiment. It cannot be defined before an examination of

the data.

Take, for example, the problem of noise. The data can be perturbed by some kind of noise. An analysis is able to define the frequency. One can then construct a filter and apply it to the data.

This step is certainly the most complex, the longest and it will be difficult to apply it systematically to all the data.

DESIRABLE QUALITIES OF A COMPUTER FOR USE IN THE EXPLOITATION OF THE DATA

The data from the satellite are always "packed" on a magnetic tape. Therefore bits must be manipulated for each data point. With such a large volume of data, it is desirable that the machine cycle be rapid.

The method employed depends on the machine to be used. At GSFC, the machines most used at this time are the IBM 7094 Model II, and Univacs 1107 and 1108. Other computers, such as the IBM 7010 and IBM 1410 are also used in various phases of data reduction.

Desirable qualities for a computer include:

- 1. Rapid cycle
- 2. Large memory
- 3. Numerous tape drives
- 4. Rapid Input/Output
- 5. Developed software (plot programs, etc.)

An experiment yielding a large volume of data (such as OGO) would require an average of 5 to 10 hours a week, depending on the type of computer and obviously dependent on the type of operations performed on the data. Most of the time, the program of Data Reduction would be run as a single block on the computer. The rest of the time it would be used for analysis programs, plots, etc.

It is also necessary to display the data for visual examination. It is not the question of listing all or part of the data on paper. The amount of data listed depends on the experiment and what the experimenter wishes to see. A plotter is needed to plot the various forms of data in a rapid manner. At GSFC, the Stromberg-Carlson 4020 is used for many applications. It is not only possible to put the plots on film or on paper, but it is also possible to generate the listings on film, to compress the volume. For an experiment on OGO, 3 to 8 hours a week of plotter time seems reasonable.

In summary, one is able to say that a powerful computer as well as a rapid plotter are indispensable.

CONCLUSION

The contents of these pages are intentionally general and cannot be complete. In effect, all the factors are dependent on the experiment itself. However, whatever method is used, these same problems will be encountered, and one will be led to a similar approach in order to resolve the problem.

APPENDIX B

ORIENTATION OF THE OPEP

During the first ten days that the OPEP experiment was in progress, in particular, during the eclipse and twilight portions of orbits 8, 9, 35, 105, 106, 107, 108, 109, 110, 111, 114, 125, 126, and 127, it has been possible to detect two stars. Sirius and Canopus.

Using the attitude-orbit tape of the satellite, and assuming the ideal attitude for the satellite, it has been possible to calculate the "sidereal hour angle" and the declination of these two stars.

Method No. 1

This consists of using the coordinates of the OPEP package in the geocentric coordinate system (GEI system). Let \vec{x}_e , \vec{y}_e , and \vec{z}_e , be three unit vectors in the system of reference for the OPEP. Let \vec{i} , \vec{j} , \vec{k} , be three unit vectors in the system of reference for geocentric system. See Figures 1 and 2.

At the time, t, for an angle of view α , ($\alpha = 0$ at horizontal), let I_{α} be the unit vector in the direction of sight for the origin, one can write (see Figure 2):

$$\vec{I}_{\alpha} = \cos \alpha \vec{X}_{e} + \sin \alpha \vec{Z}_{e}$$

Hence, the average direction of the field of view of the

photometer is always in the $X_e^{Z_e}$ plane. The coordinates of the $(\vec{X}_e, \vec{Y}_e, \vec{Z}_e)$ system of unit vectors are given on the attitude-orbit tape at the time t in the $(\vec{i}, \vec{j}, \vec{k})$ system of reference.

$$\vec{X}_{e} \begin{cases} (\text{proj } \vec{X}_{e}) & \vec{i} = a \\ (\text{proj } \vec{X}_{e}) & \vec{j} = b \\ (\text{proj } \vec{X}_{e}) & \vec{k} = c \end{cases}$$

$$\vec{Y}_{e} \begin{cases} (\text{proj } \vec{Y}_{e}) & \vec{i} = a^{\dagger} \\ (\text{proj } \vec{Y}_{e}) & \vec{j} = b^{\dagger} \\ (\text{proj } \vec{Y}_{e}) & \vec{k} = c^{\dagger} \end{cases}$$

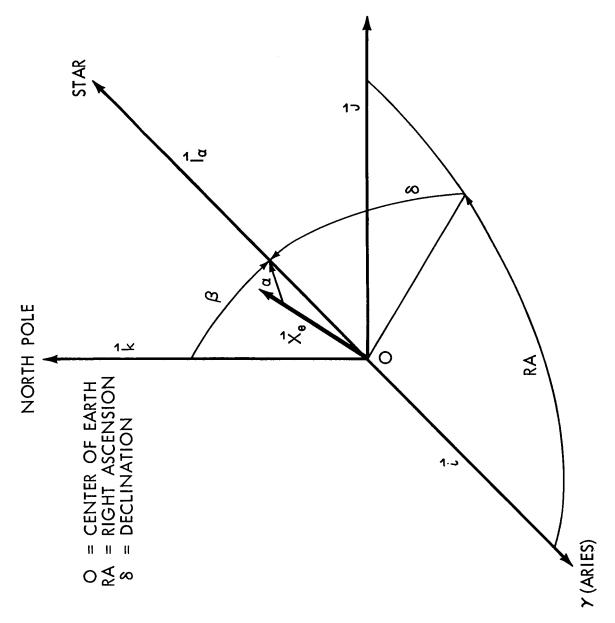


Figure 1 - Coordinate system used for Method No. 1.

Figure 2 - Definition of $\vec{I}\alpha$

$$\vec{Z}_{e} \begin{cases} (\text{proj } \vec{Z}_{e}) & \vec{i} = a'' \\ (\text{proj } \vec{Z}_{e}) & \vec{j} = b'' \\ (\text{proj } \vec{Z}_{e}) & \vec{k} = c'' \end{cases}$$

So, the coordinates of \vec{l}_{α} in the $(\vec{i}, \vec{j}, \vec{k})$ system are:

$$\vec{I}_{\alpha} \begin{cases} a \cos \alpha + a'' \sin \alpha \\ b \cos \alpha + b'' \sin \alpha \\ c \cos \alpha + c'' \sin \alpha \end{cases}$$

The coordinates of the unit vectors are:

$$\vec{i} \begin{cases} 1 & \vec{j} \begin{cases} 0 & \vec{k} \begin{cases} 0 \\ 0 & 0 \end{cases} \\ 0 & 0 \end{cases}$$
So $\beta = (\vec{k}, \vec{I}_{\alpha})$ it is immediately given by the cosines:

$$\cos\beta = \vec{k} \cdot \vec{I}_{\alpha}$$

or

 $cos\beta = c cos\alpha + c'' sinq$

 β is an angle between 0 to π and the cosines are sufficient to determine this angle in a unique manner.

The declination is related to this β by the simple relation

$$\delta = \frac{\pi}{2} - \arccos \beta$$

The right ascension is given in the same manner by the cosines

$$\cos RA = \vec{i} \cdot \vec{I}_{\alpha}$$

or

cos RA = a cos a + a'' sina

where

$$0 \le RA \le 2\pi$$

In order to determine the angle uniquely, it is sufficient to consider the projection of \vec{l}_{α} on \vec{j} . if (proj (\vec{l}_{α})) $\vec{j} \ge 0$ $0 \le RA \le \pi$

if
$$(\text{proj}(\vec{I}_{\alpha})) \vec{j} \ge 0 \quad 0 \le RA \le \pi$$

if (proj
$$(\vec{I}_{\alpha})$$
) $\vec{j} \leq 0$ $\pi \leq RA \leq 2\pi$

One then immediately obtains the "sidereal hour angle" (SHA) by the relation

$$SHA = 2\pi - RA$$

Conclusion

At the time, t, by interpolation one obtains from the attitude-orbit tape \vec{X}_e , \vec{Y}_e , and \vec{Z}_e in the $(\vec{i}, \vec{j}, \vec{k})$ system. One also has a given by the position of the photometer mirror. So one can calculate SHA and δ . One is able then to compare the calculated values with those given in tables (e.g. the Nautical Almanac).

Method No. 2

This method, more direct, is for the purpose of computing the angle, α , defined above. This uses only the vector \vec{P} giving the position of the satellite and the vector \vec{S} pointing in the direction of the star. See Figure 3.

When the photometer has a star in its field of view, one is able to define the unit vector \mathbf{I}_{α} in the direction of the observation and parallel to the unit vector, $\vec{\mathbf{S}}$, which starts from the center of the earth and points toward the star.

Using the right ascension (RA) and the declination (δ) the coordinates of \vec{S} in the geocentric system are: (see Figure 4)

$$\vec{S} = \begin{cases} \cos \delta & \cos RA \\ \cos \delta & \sin RA \\ \sin \delta \end{cases}$$

The coordinates of \vec{P} at the time, t, are known in the geocentric system. The angle would be in our case, always between 0 and π .

between 0 and
$$\pi$$
.
$$\cos \beta = \frac{\mathbf{p}}{|\mathbf{p}|} \cdot \vec{\mathbf{S}}$$

or

$$\alpha = -\frac{\pi}{2} + \beta = -\frac{\pi}{2} + \text{arc cos } (\vec{p} \cdot \vec{s})$$

One obtains then simply the angle of depression, α , which one compares with the angle given by the position of the mirror of the photometer.

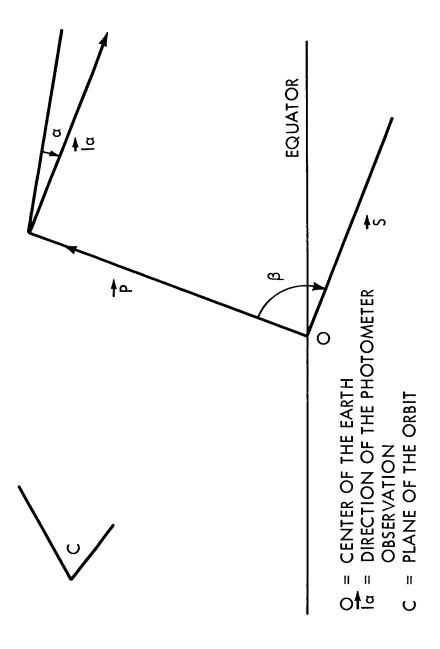


Figure 3 - Coordinate system used for Method No. 2.

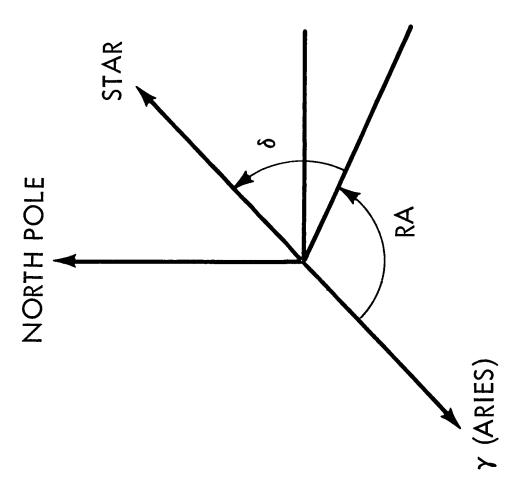


Figure 4 - Coordinates of the star.

Results

Canopus ($\delta = -52^{\circ}$ 40' 12") has appeared in the field of view of the instrument about 35 times. Sirius ($\delta = -16^{\circ}$ 39' 48") has appeared ten times.

(Sirius is more difficult to detect because of the stray light and the oscillatory movement of the OPEP container.)

APPENDIX C

REDUCTION OF OSCILLATIONS

PURPOSE

The purpose of this method is to reduce a stray oscillation in the data associated with a motion in azimuth of the OPEP container.

HYPOTHESIS

It is assumed that the principal source of stray light which comes into the photometer is the reflection of sunlight from the spacecraft, probably from the VLF antenna. The amount of stray light entering the photometer is a function both of the azimuth, Ψ , of the OPEP container, and of the mirror angle, α , the depression of the line of sight below the X-Y plane of the spacecraft.

METHOD

The amount of stray light can be represented by a family of curves as illustrated in Figure 1. Since the oscillation is of relatively small amplitude (3 degrees total amplitude) the variation of stray light as a function of angle during one oscillation can be approximated by a straight line segment.

For each value of α , there are available two successive observations of total signal from the photometer, I_T , and the corresponding values of Ψ .

The total signal can be represented by the following expression:

$$I_{T}(\alpha, \Psi) = I_{S}(\alpha) + I_{L}(\alpha, \Psi)$$
where

 $I_S = intensity of the airglow$

I_I = intensity of the stray light

One cannot compute I_L . However, one can try to correct I_T for the stray oscillation and compute $I_T(\alpha, \Psi_{ave})$.

By using Taylor's formula, and limiting it to the first order, one can write (1) as

$$I_{T}(\alpha, \Psi) = I_{S}(\alpha) + I_{L}(\alpha, \Psi_{ave}) + \Delta \Psi I_{L}^{*}(\alpha, \Psi_{ave})$$

$$\Delta \Psi = \Psi - \Psi_{ave}$$

so,

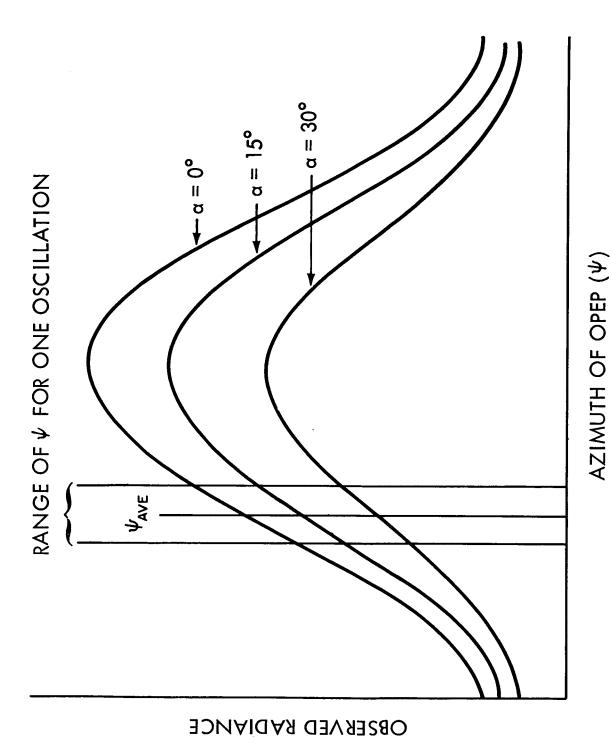


Figure 1 - Intensity of stray light as a function of OPEP azimuth and mirror position.

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$$I_{T}(\alpha, \Psi_{ave}) = I_{S}(\alpha) + I_{L}(\alpha, \Psi_{ave})$$

= $I_{T}(\alpha, \Psi) - \Delta \Psi I_{L}'(\alpha, \Psi_{ave})$

 $I_{\mathbf{T}}(\alpha, \ \forall)$ is given by the photometer

 $\Delta \Psi$ is known from the telemetered spacecraft data

 $I_L'(\alpha, \Psi_{ave})$ must be computed

For this one can use both measurements corresponding to the same step of the mirror. First, from the data corresponding to one scan of the mirror, one computes the average value of Ψ . Then one examines the data to find observations which happen to have been made when $\Psi = \Psi_{ave}$. In general, for a given mirror position, if Ψ for one data point equals Ψ_{ave} , then Ψ for the other data point is not equal to Ψ_{ave} . That is,

data point 1 α , Ψ_{ave} data point 2 α , $\Psi \neq \Psi_{ave}$

For data point 1, one can write

$$I_L(\alpha, \Psi_{ave}) = I_L(\alpha, \Psi_{ave}) + 0$$

For data point 2,

 $I_L(\alpha, \Psi) = I_L(\alpha, \Psi_{ave}) + \Delta \Psi I_L'(\alpha, \Psi_{ave})$ Noting that the first term on the right hand side is data point 1,

$$I_L'(\alpha, \Psi_{ave}) = I_L(\alpha, \Psi - I_L(\alpha, \Psi_{ave}))$$

or

$$I_L'(\alpha, \Psi_{ave}) = \frac{\text{data point } 2 - \text{data point } 1}{\Delta \Psi}$$

 $I_S(\alpha)$ being the same for data point 2 and data point 1. So for a scan, we can obtain by this means, discrete values of $I_L'(\alpha, \ _{ave})$.

Since the source of stray light is somewhat above the photometer we add the hypothesis that the amplitude of the oscillation is a linear function of α . That is,

$$I_{L}'(\alpha, \Psi_{ave}) = A\alpha + B$$

To take into account the errors of measurement, we will compute A and B by a least squares method. Finally, the correct values will be given by

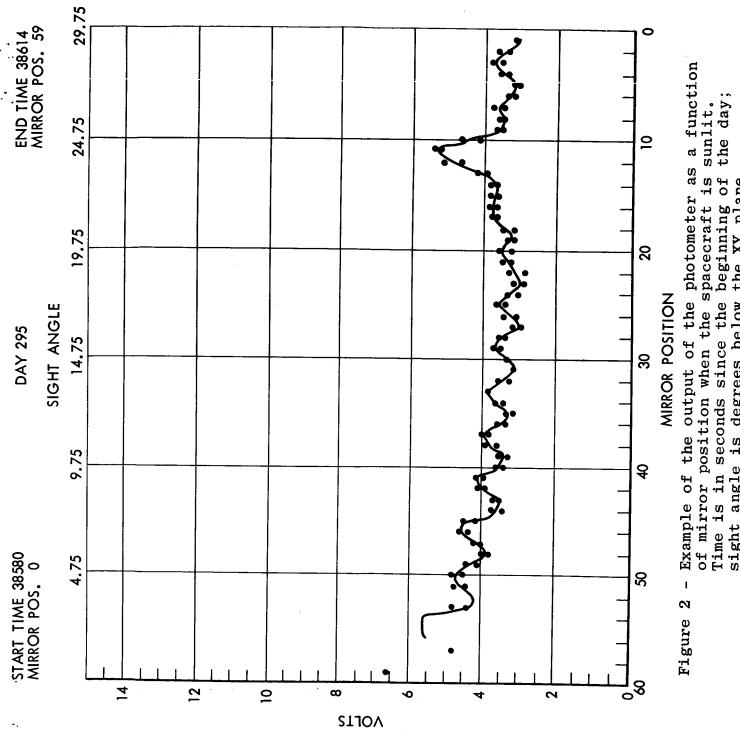
$$I(\alpha, \Psi_{ave}) = I_T - \Delta \Psi (A\alpha + B)$$

UTILIZATION OF THE DATA

The block of data used to compute A and B has been chosen to be a single scan of the mirror to satisfy the hypothesis on the amplitude of the oscillations. This corresponds to a maximum of 120 data points. This scan represents about 36 seconds of observations. This is a long enough period to give several pairs of points for computation of A and B, but short enough so that effects of temporal variations of airglow and of Y are minimized.

CONCLUSION

Figure 2 and Figure 3 are the result of the application of this method. The method is satisfactory when the data are not too noisy.



- Example of the output of the photometer as a function of mirror position when the spacecraft is sunlit.
Time is in seconds since the beginning of the day; sight angle is degrees below the XY plane.

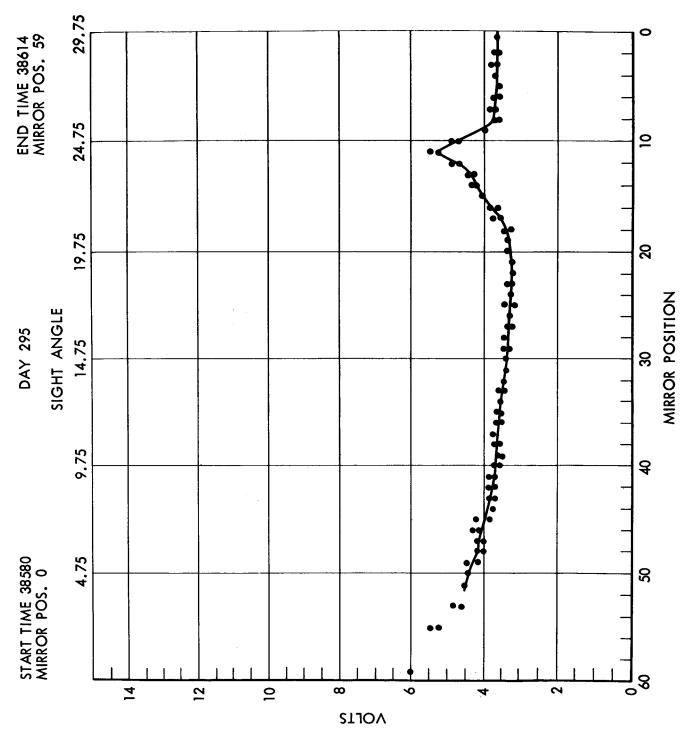


Figure 3 - Same data as in Figure 2 after processing to remove oscillations due to OPEP rotation.

APPENDIX D

COMPUTATION OF THE VERTICAL EMISSION PROFILE

STATEMENT OF THE PROBLEM

The photometer looks in different directions through the airglow layers, scanning an angle of 30° by steps of 1/2 degree from the horizontal plane of the spacecraft.

It measures the integrated emission along the line of sight. Using these data, emission profile versus altitude of the 6300A emission line was computed.

The photometer has a field of view, Ω . In a specific direction, at a distance $\{r\}$ from the instrument, the rate of photon emission per unit elemental volume of space is F(r). Let A be the sensitive area of the photometer, \vec{r} being normal to A.

The rate at which photons fall upon the receiving area from an elemental volume of the sky of length dr is $(A/4\pi r^2)F(r)dr$.

The rate at which all the photons emitted by the sky are received by the photometer is

$$\mathbf{J} = \int_{0}^{\infty} (\mathbf{A}/4\pi \mathbf{r}^{2}) \Omega \mathbf{r}^{2} \mathbf{F}(\mathbf{r}) d\mathbf{r}$$

or

$$J = \frac{A\Omega}{4\pi} \int_{0}^{\infty} F(r) dr$$

To put J into units of rayleighs, with r in cm and F(r) in photons cm⁻³sec⁻¹

(1)
$$J = 10^{-6} \int_{0}^{\infty} F(r) dr$$

$$= k \int_{0}^{\infty} F(r) dr$$

The unknown is the function F(r); J is the light intensity measured by the photometer.

MATHEMATICAL EQUATIONS

Replace (1) by
(2)
$$J = k \int_{zs}^{\infty} I(z) \frac{dr}{dz} dz$$

where zs = altitude of the spacecraft

I(z) = vertical emission versus altitude

For all these computations, assume the earth to be locally a sphere, and an ideal attitude of the spacecraft. the unit vector, n, which points from the photometer towards the center of the earth, and r, along the line of sight, are both in the plane of the orbit. (see Fig. 1)

To compute dr/dz.

In the triangle, oab
$$(3) z = [r^2 + (R + zs)^2 - 2r(R + zs) \sin \alpha] - R$$

$$\frac{dz}{dr} = \frac{r - (R + zs) \sin \alpha}{r^2 + R}$$

Hence from (2)

$$J = k \int_{ZS}^{\infty} \frac{I(z) (R + z)}{r - (R + zs) \sin\alpha} dz$$
Eliminating r. From (3)

$$r^2 - 2r(R + zs) sin\alpha - (R + z)^2 + (R + zs)^2 = 0$$

Hence,

$$r = (R + zs) \sin\alpha \pm [(R + z)^{2} - (R + zs)^{2}\cos^{2}\alpha]^{1/2}$$

$$J = -k \int_{zs}^{zmin} \frac{I(z) (R + z) dz}{[(R + z)^{2} - (R + zs^{2}\cos^{2}\alpha]^{1/2}} + \int_{zmin}^{\infty} \frac{I(z) (R + z) dz}{[(R + z)^{2} - (R + zs)^{2}\cos^{2}\alpha]^{1/2}}$$

Making the hypothesis that I(z) is a function only of z,

$$J = 2k \int_{zmin}^{zs} \frac{(R+z) I(z)dz}{[(R+z)^2 - (R+zs)^2 \cos^2 \alpha]^{1/2}} + \int_{zs}^{\infty} \frac{(R+z) I(z)dz}{[(R+z)^2 - (R+zs)^2 \cos^2 \alpha]^{1/2}}$$

with $k = 10^{-6}$

Restriction upon a

 $zmin = (R + zs) cos\alpha - R$

integrating from a minimum altitude, zmini, therefore, zmin ≥ zmini ≥ 0

and

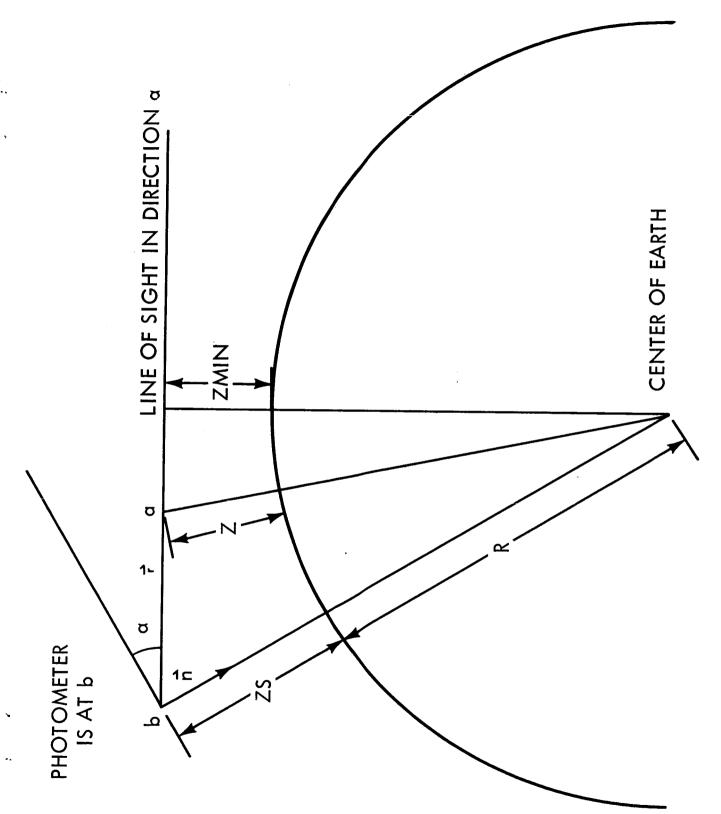


Figure 1 - Geometry of the problem in the plane of the orbit.

$(R + zs) \cos \alpha - R \ge zmini$

Solution of the problem by means of "step function"

Recall the hypotheses:

- 1. I(z) is a function of z only
- 2. The spacecraft attitude is ideal
- 3. Earth is locally a sphere
- 4. The photometer is integrating along the line of sight The data consisted of a reading of the instrument every 288 milliseconds along with the direction of the line of sight and the altitude of the photometer. For the set of data, J_i , $j = 1, \ldots$.

$$J_{j} = 2k \int_{zmin(j)}^{zs(j)} \frac{(R+z) I (z) dz}{[(R+z)^{2} - (R+zs(j))^{2}cos^{2}\alpha(j)]} \frac{1/2}{[(R+z)^{2} - (R+zs(j))^{2}cos^{2}\alpha(j)]}$$

$$+k \int_{zs(j)}^{\infty} \frac{(R+z) \ I(z) \ dz}{[(R+z)^2 - (R+zs(j))^2 \cos^2 \alpha(j)]^{1/2}}$$

Choose a series of altitudes, z_1 , z_2 , z_3 ... z_N , $z_N + 1$. Between two altitudes, z_i , z_{i+1} assume that

 $I(z_1) \le I(z)$ $I(z_{i+1})$ equals a constant. Under z_1 and above z_{N+1} , I(z) = 0.

Thus the curves are approximated by a series of steps as shown in Fig. 2. This linearized the problem. Let

$$B = \int_{z_{i}}^{z_{i+1}} \frac{I(z) (R + z) dz}{[(R + z)^{2} - R + zs)^{2} \cos^{2}\alpha]^{1/2}}$$

$$B = I(z_{i}) \left[\left[(R + z)^{2} - (R + zs)^{2} \cos^{2} \alpha \right]^{1/2} \right]_{z_{i}}^{z_{i+1}}$$

Let

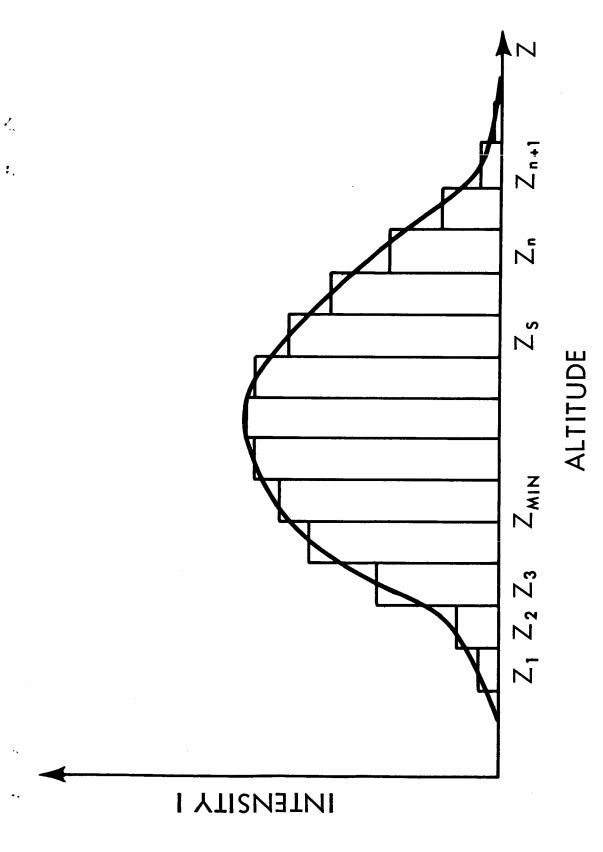


Figure 2 - Nomenclature of the layers as a function of altitude.

$$\varphi(z) = [(R + z)^2 - (R + zs)^2 \cos^2 \alpha]^{1/2}$$

(4)
$$B = I(z_i) [\varphi(z_i + 1) - \varphi(z_i)]$$

For each data point, the coefficient of $I(z_1)$ for i=1,...N is computed. Call $J_j(\alpha, z_s)$ the measurement corresponding to an altitude of the photometer of $z_s(j)$ and an angle of $\alpha(j)$. Call I_1 , $I_2,...,I_N$ the values of I(z) between (z_1, z_2) , $(z_2, z_3),...(z_N, z_{N+1})$.

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 $zmin(j) = (R + zs(j))cos\alpha(j) - R$ is the minimum altitude of the beam over the surface of the earth.

Suppose for this data

$$z_1 \langle z_2 \rangle \langle \cdots \langle z_h \rangle z_{m+1} \rangle \langle z_{m+1} \rangle \langle \cdots \rangle \langle z_{n+1} \rangle \langle \cdots \langle z_{n+1$$

Integrating from zmin(j) to z_{N+1} using relation (4)

$$J_{j}(\alpha,zs) = k \begin{bmatrix} h-1 \\ \Sigma \\ i=1 \end{bmatrix} 0 \ I_{i} + 2 \ I_{h} \left[\phi_{j}(z_{h+1}) - \phi_{j}(z_{min}(j)) \right] + 2 \begin{bmatrix} i=e^{2}-1 \\ \Sigma \\ i=h+1 \end{bmatrix} I_{i}(\phi_{j}(z_{i+1}) - \phi_{j}(z_{i})) + 2 \begin{bmatrix} i=e^{2}-1 \\ i=h+1 \end{bmatrix} I_{i}(\phi_{j}(z_{i+1}) - \phi_{j}(z_{i})) + 2 \begin{bmatrix} i=N \\ i^{2}=e^{2}+1 \end{bmatrix} I_{i}[\phi_{j}(z_{i}+1) - \phi_{j}(z_{i})]$$

From the physical point of view, that means there is no contribution of layers 1, 2,...h-1, and a different contribution for each of the other layers, depending upon the path length through that layer (each layer corresponds to a step of the function).

(5)
$$J_j(\alpha,zs) = \sum_{e=1}^{N} k A_{h,j} I_e j = 1,...n$$

with for $h \ge 2$

$$\begin{aligned} &A_{ej} = 0 \quad \text{for } e = 1, \dots h-1 \\ &A_{hj} = 2 \left[\varphi_{j}(z_{h+1}) - \varphi_{j}(z_{min}(j)) \right], \ z_{h} \leq z_{min}(j) \leq z_{h+1} \\ &A_{ej} = 2 \left[\varphi_{j}(z_{e+1}) - \varphi_{j}(z_{e}) \right], \ e = h+1, \dots e'-1 \\ &A_{e'j} = \left[\varphi_{j}(z_{s}(j)) + \varphi_{j}(z_{e'+1}) - 2\varphi_{j}(z_{e'}) \right], \ z_{e'} \leq z_{s}(j) \leq z_{e'+1} \\ &A_{ej} = \left[\varphi_{j}(z_{e+1}) - \varphi_{j}(z_{e}) \right], \ e = e'+1, \dots N \end{aligned}$$

For the set of data, j = 1, ...n, there are N unknowns, I_1 , I_2 ,... I_N , with n >> N. This linear system with more equations than unknowns is solved by a least squares method. Minimizing

$$M = \sum_{j=1}^{n} (J_j - k_{e=1}^{N} A_{ej} I_e)^2$$
, k being a constant.

Taking the partial derivatives with respect to the unknowns I_i and setting = 0 gives the optimum value of M.

$$\frac{\delta \mathbf{M}}{\delta \mathbf{I_i}} = 0 \qquad \text{for } \mathbf{i} = 1, \dots N$$

This leads to

or

(6)
$$\sum_{j=1}^{n} A_{ij} = k \sum_{j=1}^{n} A_{ij} = k \sum_{e=1}^{n} A_{ej}$$

Substituting (5) into (6) gives the linear system

This linear system was solved by a classical method (Gauss) of inverting the matrix of the coefficients \mathbf{A}_{ij} to get the parameters I.. \mathbf{I}_e .. \mathbf{I}_N , the vertical emission in rayleighs of the layer e.

CONSIDERATIONS ABOUT THE METHOD AND CHOICE OF THE MODEL

The preceding pages have described the mathematical method used and the resulting set of equations. It was desired to compute the vertical distribution of airglow between about 50km to 450km. (These limits were chosen after initial study of the data). The field of view of the instrument is such that for spacecraft altitudes above 400km, the minimum meaningful layer thickness is about 15km. This indicates a maximum number about 27 parameters.

The least squares method reduced the problem from a linear system with more equations than unknowns to a square matrix. By inverting the square matrix supplied by the least squares procedure the 27 parameters are computed. However, unless care is taken in the choice of the model used to fit the curve, in this case the choice of the layers, the matrix supplied by the least squares method may be singular and hard to invert.

After computing the coefficients of the system without regard for this fact, one will frequently find when trying to invert this matrix that it is singular. Even a double precision computation will give a determinant of zero. In fact the product of the matrix of the coefficients by its invert will give a result not very close to the unit matrix. If one looks at these coefficients, he will find almost proportional equations.

To break this singularity of the matrix, one possibility is to give to each parameter an important contribution in at least one equation. In other words, there must exist for each I_1 parameter, an equation such that $A_{1,i}^{>>}A_{i,j}$, $i\neq 1$.

For the single scan method, the model was selected such that the preceding condition would exist by fixing the thickness of the layers equal to the maximum difference in minimum altitude reached of any two adjacent lines of sight. For the parallel method the layers were of different thicknesses but care was taken to assure that at least one line of sight reached

its minimum altitude within each layer chosen. The mathematical results of this determination of the model is that the coefficients of the main diagonal and of the adjacent subdiagonals have the greater magnitude for each row. The matrix in this case is well conditioned and easy to invert. The problems of instability, in this particular case, disappear.

From the physical point of view that means that each layer has at least one measurement such that the path length through this layer is the greatest. This determines the choice of the Z_i. In practice this has meant that there were between 15 and 20 layers chosen most of the time. See Fig. 3.

RESULTS

This method, relatively simple, has led to good results, especially when the hypothesis of uniformity could be applied to the phenomena. This has been the case except around the auroral zone, which is far from being uniform. In the case of layers of weak emission, however, some small negative values were computed. This has not been a problem because it occurred at an altitude less than the one of interest. In this case, these negative values do not affect the other parameters. The accuracy depends more on the photometer. In our case we could not go under a step corresponding to a thickness of 20km.

In the case of non-uniform emission, there is need for improvement.

AB >> EF OR EA OR BC or CD

Figure 3 -Diagram to illustrate the criteria for selection of layers.